Performance Testing of a Thermally Gradient Tetralattice^{TM} Heat Fin Built via Solid Freeform Fabrication

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Abstract

Solid Freeform Fabrication (SFF) is a powerful technology that allows the creation of custom designs, which would be impossible to build using conventional fabrication techniques, become a reality. Several heat transfer devices have been custom designed and manufactured using solid freeform fabrication at the Milwaukee School of Engineering research facilities by prior NSF-REU participants. One of these heat transfer devices is a thermally gradient cylindrical heat fin whose structure incorporates an extremely complex gradient Tetralattice^{TM} geometry. The goal of this project was to test the performance characteristics of this unique heat transfer device and compare and contrast them to those of conventional cylindrical designs that are used in a variety of engineering applications. The experiments performed involved the use of steady-state conduction and convection principles that shed light into the thermodynamic abilities of this new structure. A detailed understanding of the dynamics and capabilities of this heat transfer device is important as it has the potential to both exceed the performance of conventional heat fins and provide a wide range of applications in the aerospace, automotive, heating, and cooling industries.

1. Introduction

Heat fins are heat transfer devices that facilitate the transfer of heat from one location to another. Their most frequent application is one in which an extended surface is used specifically to enhance heat transfer between a solid and an adjoining fluid [1]. These devices can be found in a number of engineering applications where excessive amounts of heat are undesirable. All heat fins work in accordance with the laws of thermodynamics. These laws are made manifest in a heat fin by three unique principles known as conduction, convection, and radiation.

1.1 conduction

Conduction is a material’s inherent ability to distribute heat within itself from an area of high concentration to an area of lower concentration. The rate at which this distribution occurs is known as the thermal conductivity of the material. Substances that have a high thermal conductivity are classified as conductors. Furthermore, substances that have a low thermal conductivity are classified as insulators. All known materials possess an amount of thermal conductivity and, consequently, there is now such thing as a perfect insulator. The equation that governs heat transfer through conduction in a heat fin is known as the law of thermal conduction [2] and is expressed as

$$\dot{Q} = kA_c\left|\frac{dT}{dx}\right| \tag{1}$$

where $\dot{Q}$ is the amount of heat transfer, *k* is the thermal conductivity of the material, A_c is the cross-sectional area of the heat fin, and *dT/dx* is the variation of temperature with position.

1.2 convection

Convection occurs when a low temperature fluid makes contact with a solid that has a higher temperature and absorbs heat from the solid. The term fluid applies to any type of gas or liquid. This process can be divided into two types: free convection and forced convection. Free convection occurs when the fluid flow is caused by natural means, such as density differences in the air. Forced convection occurs when the fluid flow is caused by an external source such as a fan or a pump. The equation that governs heat transfer through convection in a heat fin is known as Newton's law of cooling [3] and is expressed as

$$\dot{Q} = hA_s\left(T - T_\infty\right) \tag{2}$$

where $\dot{Q}$ is the amount of heat transfer, *h* is the thermal convection coefficient, A_s is the heat fin surface area, *T* is the temperature of the heat fin at a particular location, and T_∞ is the temperature of the fluid.

1.3 radiation

Radiation is an object's ability to reflect and absorb electromagnetic waves. All objects radiate energy continuously in the form of electromagnetic waves produced by thermal vibrations in its constituent molecules [2]. A body that is hotter than its surroundings radiates more energy than it absorbs, whereas a body that is cooler than its surroundings absorbs more energy than it radiates. The equation that governs heat transfer through radiation in a heat fin is known as Stefan's law and is expressed as

$$\dot{Q} = \sigma e A_s (T^4 - T_\infty^4) \tag{3}$$

where $\dot{Q}$ is the amount of heat transfer, σ is the Stefan-Boltzmann constant, *e* is the heat fin's emissivity constant, A_s is the heat fin surface area, *T* is the temperature of the heat fin at a particular location, and T_∞ is the temperature of the heat fin's surroundings. In the case of a heat fin, the surroundings temperature is the temperature of the fluid that it makes contact with.

2. Solid Freeform Fabrication

Solid Freeform Fabrication (SFF) is a technology that enables the fabrication of custom-designed objects with unique properties directly from computer data. The basic operation of any SFF system consists of slicing a three-dimensional computer model into thin cross sections, translating the result into two-dimensional position information, and using this data to control the placement of solid material. This process is repeated for each cross section and the object is built up one layer at a time [4].SFF is also known as additive manufacturing because material is added to a design to produce an object as opposed to conventional fabrication techniques where material is subtracted in order to produce a final product.

2.1 selective laser melting

Selective laser melting (SLM) is a solid freeform fabrication tool that has the capability to produce 100% dense metal parts from customary metal powders [5]. SLM works by inputting a three-dimensional computer-aided design file into a computer. The SLM software divides the three-dimensional design into layers of 30 μm thickness and then sends this information to an infrared laser. The laser traces the layer's geometry and melts stainless steel powder where it is needed until the layer is finished. Another layer is then started above it and the process is repeated until the design is fully manufactured. Figure 1 shows an example of a selective laser melting apparatus.

Figure 1 Selective laser melting apparatus

Figure 2 Gradient Tetralattice^{TM} heat fin

2.2 gradient Tetralattice^{TM} heat fin

A thermally gradient Tetralattice^{TM} heat fin is a unique design that is shaped in the form of a cylindrical rod with a radius of 0.75 inches and a length of 7.5 inches. The cylinder is composed of small Tetralattice^{TM} units that imitate the shape taken by covalently bonded carbon atoms that exist in diamond. The fin is 100% solid on one end and decreases in material density in an approximately linearly gradient fashion along its length. As stated earlier, heat fins are used to remove excessive amounts of heat from a place where heat is not desired. A great heat fin is one that has a high thermal conductivity and a large amount of surface area exposed to a fluid. A large amount of thermal conductivity allows the fin to conduct a significant amount of heat from the heat source. Likewise, a large surface area allows the fin to transfer more of this heat to the fluid that surrounds it. One of the triumphs of a gradient Tetralattice^{TM} heat fin is that it exposes a large amount of surface area to a fluid within a given volume. Since surface area is directly related to convection, an increase in surface area also augments the amount of heat released into a fluid by the gradient heat fin. For the purpose of this research, the fluid in question was air flowing through the fins through free convection. Figure 2 shows a picture of the thermally gradient Tetralattice^{TM} heat fin. The complex geometry of this design makes its manufacture through conventional means impossible. Because of this, the gradient heat fin was built out of stainless steel using selective laser melting.

2.3 research objectives

The objective of this research was to test the performance characteristics of this unique heat transfer device and compare and contrast them to the characteristics of cylindrical designs of the same material and dimensions that are commonly used. In order to accomplish this, a suitable experiment had to be developed that had the capability to measure the performance of both the conventional and the gradient Tetralattice^{TM} heat fins. The experiments performed on these devices involved steady-state conduction and convection principles that shed light into the behavior of the gradient device.

3. Heat Fin Theory

When a cylindrical heat fin of constant cross-sectional area working at steady-state is considered, a differential equation that describes its conductive and convective properties can be derived by the use of a differential heating element. The final form of the differential equation is expressed as

$$\frac{d^2T}{dx^2}+\left(\frac{1}{A_c}\frac{dA_c}{dx}\right)\frac{dT}{dx}-\left(\frac{1}{A_c}\frac{h}{k}\frac{dA_s}{dx}\right)\left(T-T_\infty\right) \tag{4}$$

where A_c is the fin's cross-sectional area, A_s is the fin's surface area, h is the thermal convection coefficient, k is the fin's thermal conductivity, x is the distance from the heat source, T is the temperature at a given location on the fin, and T_∞ is the temperature of the air. The assumptions made to derive this equation were the following:

- Constant cross-sectional area
- One-dimensional conduction along the length of the heat fin
- Constant thermal conductivity throughout the fin
- Radiation is negligible
- Thermal convection coefficient is uniform over the fin's surface.

The general solution to this differential equation is expressed as

$$T-T_\infty = C_1 e^{x\sqrt{hp/kA_c}} + C_2 e^{-x\sqrt{hp/kA_c}} \tag{5}$$

where p is the perimeter of the heat fin and C_1 and C_2 are constants of integration. Assuming that convection heat transfer occurs at the tip of the heat fin and defining a new variable θ (labeled excess temperature, where $\theta = T\text{-}T_\infty$), a final expression for the temperature distribution within a cylindrical heat fin can be expressed as

$$\frac{\theta}{\theta_b}=\frac{\cosh\left[m(L-x)\right]+\left(\frac{h}{mk}\right)\sinh\left[m(L-x)\right]}{\cosh(mL)+\left(\frac{h}{mk}\right)\sinh(mL)} \tag{6}$$

where θ_b is the difference of the base temperature and the temperature of the air, L is the length of the heat fin, and m is expressed as

$$m=\sqrt{\frac{2h}{rk}} \tag{7}$$

where r is the radius of the cylindrical heat fin.

The heat transfer rate for the entire fin can also be determined from equation (6) and is expressed as

$$q_f = M\frac{\sinh(mL)+\left(\frac{h}{mk}\right)\cosh(mL)}{\cosh(mL)+\left(\frac{h}{mk}\right)\sinh(mL)} \tag{8}$$

where M is expressed as

$$M=\theta_b\sqrt{2hk\pi^2r^3} \tag{9}$$

The inherent power of the equations just defined lies in the fact that all of the variables that constitute them can be easily measured or can be found in tables or books. One variable, however, the thermal convection coefficient, *h*, can only be determined through empirical relationships or through experiment. This variable was computed through the use of both methods and the results agreed with one another.

4. Experimental Procedure

A Hewlett-Packard 34970A Data Acquisition System was used to take temperature readings in each experiment. These readings were taken through the use of thermocouples constructed from K-wire. A Corning PC-420 Stirrer/Hot Plate was used as the source of heat. Three heat fins were tested: a solid stainless steel model, a hollow-centered stainless steel model nicknamed the "Holey Cow", and the thermally gradient Tetralattice^{TM} heat fin. Each heat fin had a length of 7.5 inches, a radius of 0.75 inches, and was divided into six different planes that were 1.5 inches apart from each other. Since the assumption of one-dimensional heat transfer was made on the solid and Holey Cow fins, only one thermocouple was placed in each plane. Three thermocouples were place on each of the six planes of the Tetralattice^{TM} heat fin in order to account for possible two or three-dimensional heat transfer effects that could be present during the experiments. The hot plate was heated to its maximum capacity in order to maintain a level of consistency in the amount of heat delivered to each fin. Each experiment lasted four hours in order to allow each heat fin to reach a satisfactory steady-state temperature. The data acquisition system recorded the temperatures measured by each thermocouple every two minutes. Figure 3 shows the final experimental setup used on all three heat fins.

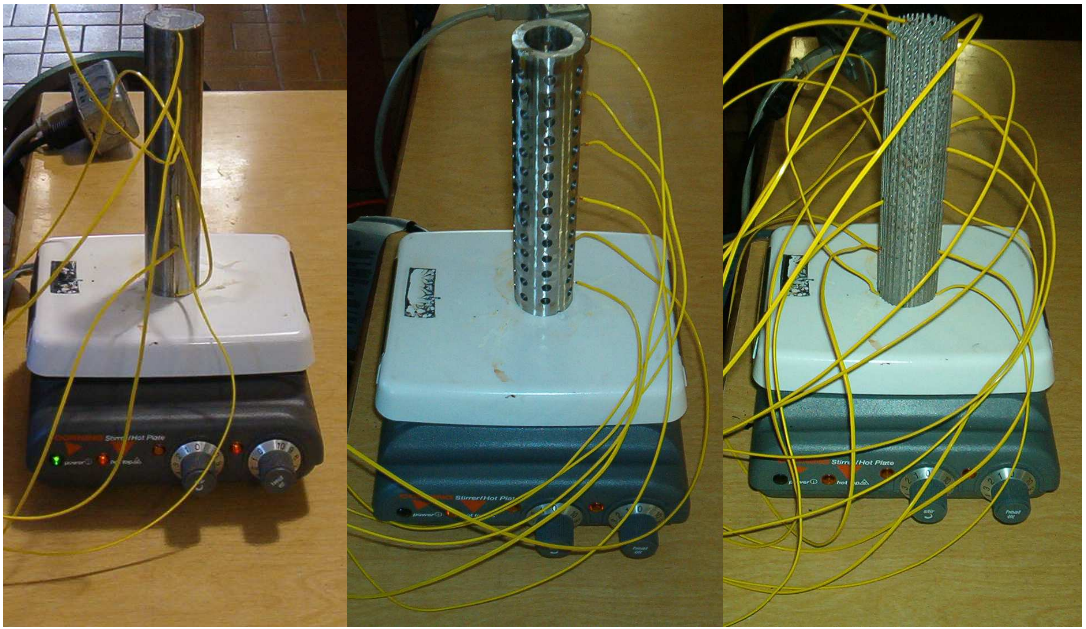

Figure 3 Solid, Holey Cow, and Tetralattice^{TM} heat fin experimental setups

5. Data/ Experimental Results

Figure 4 shows the temperature distribution (θ/θ_b) of the heat transfer test conducted on the solid stainless steel heat fin and compares it to the theoretical results of a solid heat fin of the same material and dimensions (x-axis is non-dimensional length, x/L). A free convection coefficient of 8 W/m^2 K was used in the analysis which is appropriate for these experimental conditions. The experimental results match the theory quite well. Similar tests were conducted for solid fins made of copper and aluminum which yielded results that matched the theory.

Figure 5 shows the experimental temperature distributions of the three heat transfer tests conducted on the Holey Cow heat fin. Similarly, figure 6 shows the experimental temperature distributions of the three heat transfer tests conducted on the Tetralattice^{TM} heat fin.

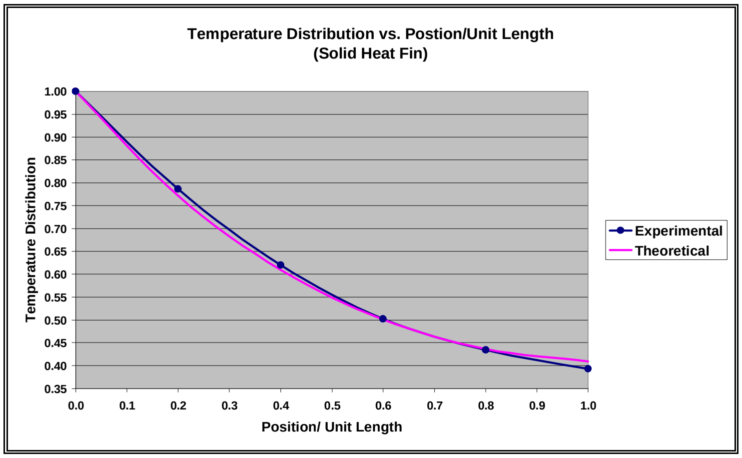

Figure 4 Temperature Distribution of solid heat fin

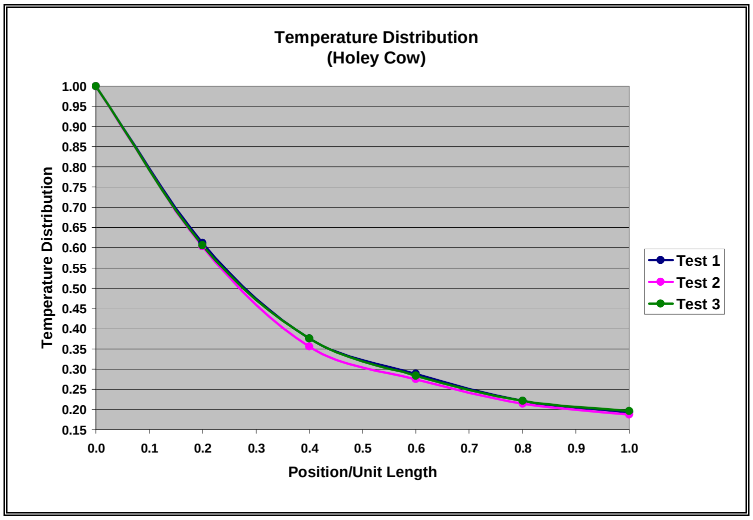

Figure 5 Temperature Distribution of Holey Cow heat fin

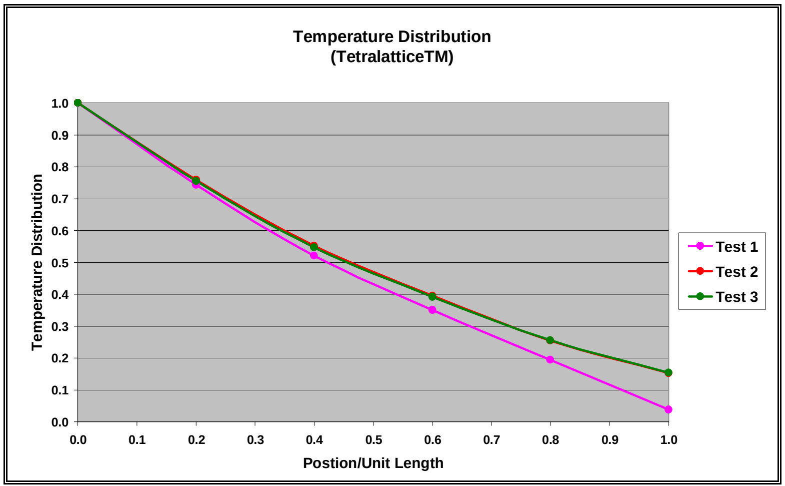

Figure 6 Temperature Distribution of Tetralattice^{TM} heat fin

Figure 7 compares the temperature distribution of the solid stainless steel heat fin and the average temperature distributions of the Holey Cow and the thermally gradient Tetralattice^{TM} heat fin.

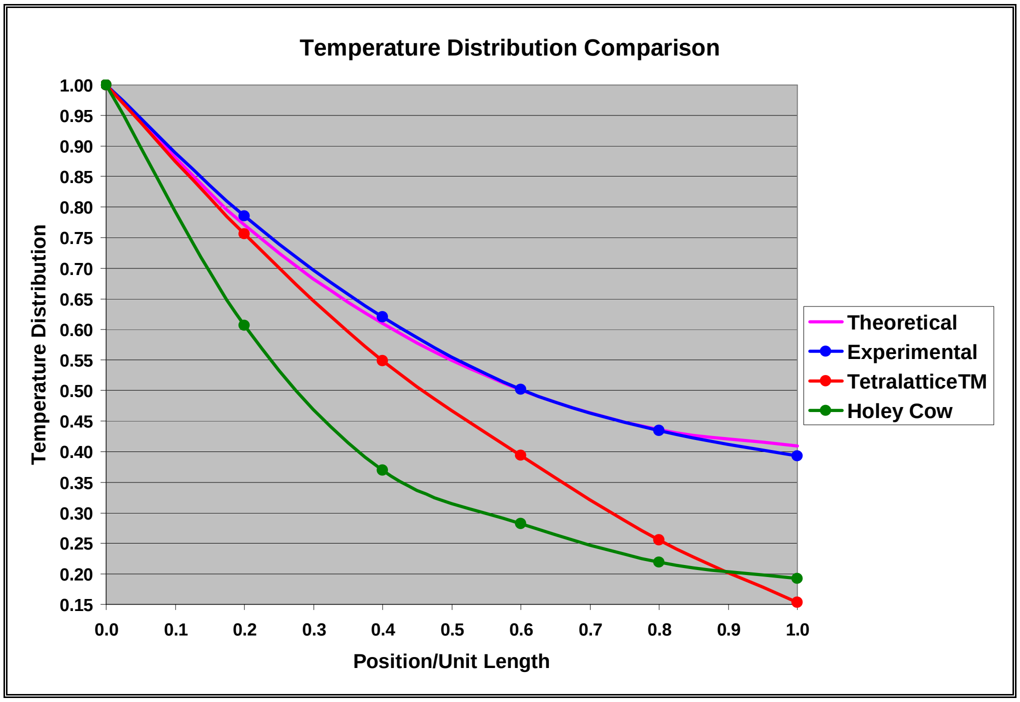

Figure 7 Comparison of Temperature Distribution in heat fins

6. Discussion of Results

The small amount of error observed between the experimental and theoretical temperature distributions of the solid stainless steel heat fin was strong evidence that the experimental procedure used throughout this research was accurate and the one-dimensional assumption was validated. Moreover, the consistent readings recorded in the temperature distribution of the three tests conducted on the Holey Cow heat fin were strong evidence that the experimental procedure used throughout this research was precise.

The author observed that the Tetralattice^{TM} heat fin released a considerable amount of smoke while it underwent its first heat transfer experiment. It was concluded that the smoke was produced when left over resin, or some other by-product formed by the selective laser melting process, was heated to the point of evaporation. The author concluded that this smoke augmented the heat transfer characteristics of the gradient heat fin. This hypothesis was verified by the second and third test where no smoke was present and the gradient device did not perform at the same level as the first test.

Due to their consistency, the results of the experiments performed on the Holey Cow were averaged when compared to the experimental results of the solid stainless steel heat fin. The same procedure was used to compare the experimental results of the gradient heat fin, but only the second and third test were taken into account. The heat transfer characteristics of the Tetralattice^{TM} heat fin were assumed to be one-dimensional because the largest temperature difference within a plane was only six Kelvin.

When the full length of each heat fin was considered, the thermally gradient Tetralattice^{TM} heat fin outperformed the solid stainless steel heat fin and the Holey Cow. The gradient heat fin also had the largest drop in temperature between its base and its other end. As of the writing of this paper, the author was attempting to quantify the heat transfer rate of both the Holey Cow and the gradient heat fin. Knowledge of these quantities will shed light into the amount of heat conducted from the hot plate by these two heat transfer devices. These results will then be compared with the heat transfer rate of the solid stainless steel heat fin.

7. Conclusion

A heat transfer experiment has been successfully established to study the temperature distribution of various fins. The analytical solution for a solid heat fin has been verified by the experiment for a solid stainless steel fin and solid fins of known thermal conductivity such as copper and aluminum. More over, two other innovative fins were tested and the Tetralattice^{TM} showed excellent heat transfer characteristics that have tremendous potential for use in automotive applications.

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9. References

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