

Using Finite Element Analysis to Investigate Lattice Fins for Thermal Behavior

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Abstract

The need to improve thermal efficiency has never been greater. Due to the growing need, rising cost, and the environmental challenges associated with energy production, the world has become more energy conscious. Currently, a large percentage of energy created for power production, or transportation is wasted due to inefficient heat transfer in a variety of applications. Fins are often used in automobiles, motorcycles, aircraft, power-plants, and nuclear reactors to improve heat transfer, and overall system efficiency. Fin performance generally increases with its surface area to volume ratio. Large surface area to volume ratios can be achieved via a repeated tetrahedron lattice, or Tetralattice™ structure. This structure is modeled after the covalent bonding between the carbon atoms in diamond. This complex fin structure can be replicated through rapid prototyping technology and fabricated in metal via an additive manufacturing technique known as selective laser melting. However, mathematical modeling of these structures can prove to be quite difficult or impossible at times. Manufacturing and testing these fins can also prove to be quite expensive and difficult to accomplish as well. This research used finite element method to analyze fins of a variety of different geometries, in particular by varying the lattice thickness of a uniform Tetralattice™ fin. The software package *Patran*, was utilized in order to determine temperature distributions of each fin. A variety of different geometries of uniform fins were also analyzed in an attempt to improve fin design. Due to the complex geometry of the Tetralattice™ unorthodox methods were employed for fin analyses.

Keywords: Heat transfer, Tetralattice™, Finite element, Rapid prototyping

1. Introduction

Thermal energy is an extremely complex phenomenon to model. It is not perceived as well as other physical attributes in life; perhaps because temperature is not something that is easily visualized. Testing of thermal devices can be very relevant to a plethora of applications. Most modern technology depends heavily on the heat transfer, which is the transfer of thermal energy. Computers, automobiles, and power plants depend on effective heat transfer in order to maintain operation.

Thermal fins are nothing more than extended surfaces used to improve heat transfer between an object and an adjacent fluid. Fins are found in just about any applications where there is a large temperature difference and heat dispersion or absorption is imperative. Often times the degree to which a heat source or sink is able to move heat correlates directly to the overall efficiency of a system¹.

Ineffective heat transfer often accounts for some of the largest losses a system will experience in operation. Minimizing this effect could drastically improve how environmentally friendly several systems are.

Testing a system for heat transfer can prove to be relatively expensive as well as time consuming. Optimizing a system can become even more arduous. An obvious option would be to model a fin mathematically. Several

geometries of fins have generally accepted mathematical models for temperature distributions and heat transfer rates. However, as fin geometry becomes more complex, these models become useless.

Due to the complex nature of Tetralattice™ fins a different approach is needed to conduct analysis. Finite element method offers an option. In finite element analysis, structures are discretized into small sections or elements and energy balances are applied to each element based upon boundary conditions. The two major factors that affect the heat transfer of a fin are its surface area to volume ratio and the convection heat transfer coefficient. The higher both of these values are, the greater achievable heat transfer².

The convection heat transfer coefficient is affected by fluid properties, flow rates and flow types. The Tetralattice™ structure mimics the covalent carbon bonding in diamond and provides on the highest surface area to volume ratios of any known structure. One would assume that this type of structure would work great for heat transfer applications³.

1.1. objective

The objective of this research is to perform finite element analysis on a Tetralattice™ fin of uniform thickness in order to determine the temperature distribution through the fin. Thicknesses of the lattice will be varied to see its impact on heat transfer. Due to the extremely complex geometries of these fins unorthodox methods of analysis may need to be implemented in order to obtain accurate results from the software. Being able to perform finite element analysis on the Tetralattice™ fin will prove extremely beneficial in determining an ideal fin structure. Secondly full scale models of this analysis will be printed in 3D to provide a better visualization of the temperature distribution of fins of varying geometry, given the same boundary conditions, and material properties.

2. Theory

Heat transfer occurs through three different mechanisms: conduction, convection, and radiation, equations (1-3) below. These effects always occur such that energy from any one of these entering a system is equal to the sum of the energy stored within the system or expelled by it. Secondly these effects always occur such that heat flows from higher thermal energy to lower thermal energy⁴.

$$\dot{Q}_{Conduction} = kA \left(\frac{\Delta T}{L} \right) \quad (1)$$

$$\dot{Q}_{Convection} = hA_s (T_{Surface} - T_{\infty}) \quad (2)$$

$$\dot{Q}_{Radiation} = \epsilon A_s \sigma (T_{Surface}^4 - T_{\infty}^4) \quad (3)$$

Where:

\dot{Q} is the rate of heat transfer, (W) k is the thermal conductivity of a material, $\left(\frac{W}{m * K} \right)$

A is the cross sectional area of heat transfer (m^2) ΔT is the temperature gradient ($T_{High} - T_{Low}$) (K)

L is the length of heat travel (m) h is the convection heat transfer coefficient $\left(\frac{W}{m^2 * K} \right)$

A_s is the surface area of an object exposed (m^2) ϵ is the emissivity of a surface

σ is Boltzmann's constant $5.67 \cdot 10^{-8} \left(\frac{W}{m^2 * K^4} \right)$ $T_{Surface}$ is the temperature at the surface in Kelvin (K)

T_{∞} is the temperature of the surroundings in Kelvin (K)

2.1. fin equation

Steady state heat transfer for uniform fins is modeled by the following differential equation:

$$\rho C_p \frac{dT}{dt} = \frac{d^2T}{dx^2} - \left(\frac{1}{A_c} \frac{h dA_s}{k dx} \right) (T - T_\infty) \quad (4)$$

Where:

T is the temperature at a given point along the length of the fin, (K)

x is the distance from the heat source in the direction of the heat transfer (m)

A_c is the cross sectional area of the fin at a given location (m^2)

h the convection heat transfer coefficient ($\frac{W}{m^2 * K}$)

k the thermal conductivity of a material ($\frac{W}{m * K}$)

A_s is the surface area of the fin at a given increment (m^2)

T_∞ is the temperature of the surroundings in Kelvin (K)

Note when conducting steady state analysis the only relevant thermophysical property is thermal conductivity, the density (ρ), and the specific heat (C_p) of the material do not factor into the fin equation since temperature is no longer changing with time.

When modeling a simple fin several assumptions must be taken into account including:

- Constant cross-sectional area
- One-dimensional conduction along the length of the heat fin
- Constant thermal conductivity throughout the fin
- Radiation is negligible
- Thermal convection coefficient is uniform over the fin's surface.

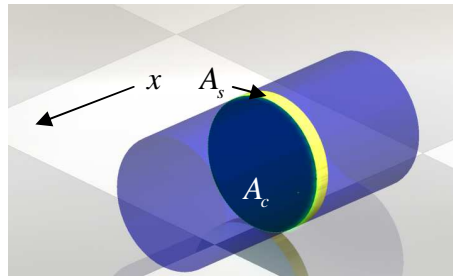


Figure 1. Visual representation of key geometric variables within the fin equation.

3. Finite Element Method

Finite element methods were developed during the 1950's by aerospace engineers in hopes to optimize aircraft. Finite element methods, finite element analysis, or FEA as it is commonly referred to takes complicated structures, and discretizes them into individual elements. This grouping of elements is called a mesh. Once a complete mesh is created, the FEA software then applies equations that pertain to the specific analysis (i.e. structural, thermal, vibrational) and solves these equations simultaneously. These solutions are often only approximations to true results but become more accurate as one creates a more fine mesh by shrinking the size of the elements within the structure.⁵

4. Tetralattice™ Structure

The Tetralattice™ structure mimics the covalent bonding that carbon atoms make within diamond. This structure has one of the highest surface areas to volume ratios of any known structure making it a desirable structure to use for in thermal fins. Due to the complex geometry of this structure, computer processing becomes extremely difficult to implement in most 3D CAD packages and FE software.

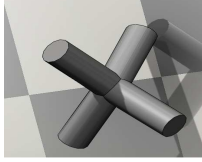


Figure 2. An individual tetrahedron lattice made in *Solidworks*.™

5. CAD Model

A thermal fin was made via *Solidworks* by building up a block of individual tetrahedrons. Once this block was built up, a cylinder of tetrahedrons was cut from it. A thin cylindrical base was mated with the bottom, for the sole purpose of creating a face for a constant boundary temperature to be applied to. This fin was constructed from tetrahedrons which had a diameter of twenty thousandths of an inch.

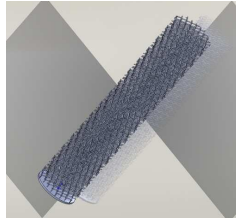


Figure 3. A uniform Tetralattice™ thermal fin.

6. Additive Manufacturing

This research project used parts created from an additive manufacturing process known as stereolithography (SLA). The process is demonstrated in the previous section Figure 3. The part itself is made in horizontal cross sections one layer at a time as a Helium-Cadmium laser scans over a UV sensitive polymer (Liquid photopolymer). After a layer is complete, the table drops and the laser scans over the liquid again creating a new layer of polymer. This process repeats for the entire thickness of the part. Once created these SLA parts are extremely fragile and weak, but built with great precision⁶.

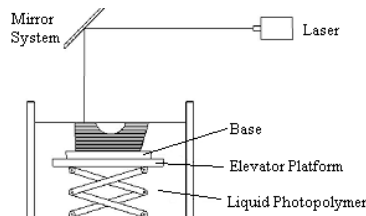


Figure 4. Stereolithography (SLA) Process.

7. Justification of Research

Additive manufacturing techniques have been implemented in previous research to fabricate a fin of gradient lattice Figure 5 thickness via a process referred to as selective laser melting (SLM). Selective laser melting is a process similar to SLA in that the part is built up layer by layer, one cross-section at a time. However instead of the laser curing layers of resin, the laser melts together powder metal in order to create the fin. The part in Figure 5 was made through this process by a German company (F & S Stereolithographietechnik GmbH). This was a very time consuming and expensive process. The part displayed in Figure 5 is a gradient Tetralattice fin 7.5" in length and 1.5" in diameter that cost over \$2000.00 USD⁷. The thermal testing was then performed on the gradient lattice fin by attaching several thermocouples to the fin Figure 6 in order to plot a temperature distribution of the fin and compare

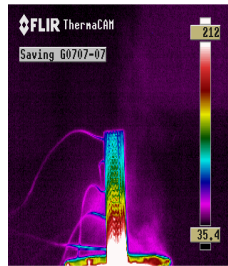
its heat transfer properties to that of a solid stainless steel of the same dimensions. The gradient fin was found to have better heat transfer properties than the solid fin. The next thermal testing was conducted using infrared thermography. This testing involved the use of a thermal imaging camera which can cost up to \$20,000.00 USD Figure 7⁸. Finite element analysis provides a definite advantage in determining the heat transfer properties of the Tetralattice structure, especially when several variations are to be studied for optimization.



Figure 5. Gradient Tetralattice™ fabricated via selective laser melting.



Figures 6. Previous research performed on a gradient Tetralattice™ fin via thermocouples.



Figures 7. Previous research performed on a gradient Tetralattice™ fin via infrared thermography.

8. Analysis and Results of FEA

In order to perform FEA on a part draw in *Solidworks*, the part was first saved in *Solidworks* as a parasolid file type. The part file was then imported into *MSC Nastran Patran*, which the author will refer to as *Patran*. After the part file was successfully imported, the part was discretized into smaller tetrahedrons by the software. After a proper mesh was applied to the part, material properties were assigned to the fin. These properties are completely arbitrary and can be applied to the part for any material. For the purpose of this research properties close to that of stainless steel were used. The major thermo physical property that *Patran* requires of the user is the thermal conductivity, for

stainless steel this value is $k = 15.15 \left(\frac{W}{m * K} \right)$. After material properties of the fin were prescribed, boundary

conditions were applied to the fin. To keep the FEA of this fin manageable for computing purposes only two major boundary conditions were applied to the fin. One was a constant 500 K temperature applied to the base of the fin,

and the other was convection to 300 K ambient for all other surfaces of the fin at a rate of $h = 10 \left(\frac{W}{m^2 * K} \right)$.

These temperatures also are relatively close to that of previously conducted research of the gradient fin. A variety of different fin geometries were explored, first of which was a cylinder which could also be modeled via accepted mathematical models equation (5). Figure 9 shows a comparison of accepted results via a mathematical model to FE

results from *Patran*. A mathematical model for non-dimensional temperature distribution for fins of uniform cross-section with a constant temperature fin base and convective fin tip is well-known:

$$\frac{\theta}{\theta_b} = \frac{\cosh[m(L-x)] + \frac{h}{mk} \sinh[m(L-x)]}{\cosh(mL) + \frac{h}{mk} \sinh(mL)} \quad (5)$$

where x is the location where temperature T is measured, L is the length of the fin, P is the perimeter of the fin, A is the cross-sectional area of the fin, k is the thermal conductivity of the fin, h is the convective heat transfer coefficient, T_b is the base temperature, T_∞ is the free-stream temperature of the surrounding air, $m = \sqrt{\frac{hP}{kA}}$, $\theta = T(x) - T_\infty$, and $\theta_b = T_b - T_\infty$.⁹

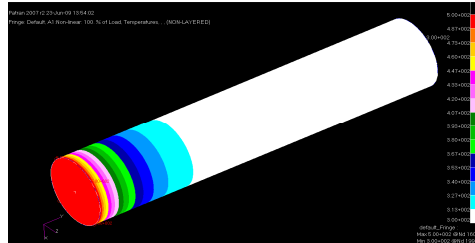


Figure 8. FE Temperature distribution Results of a simple cylinder.

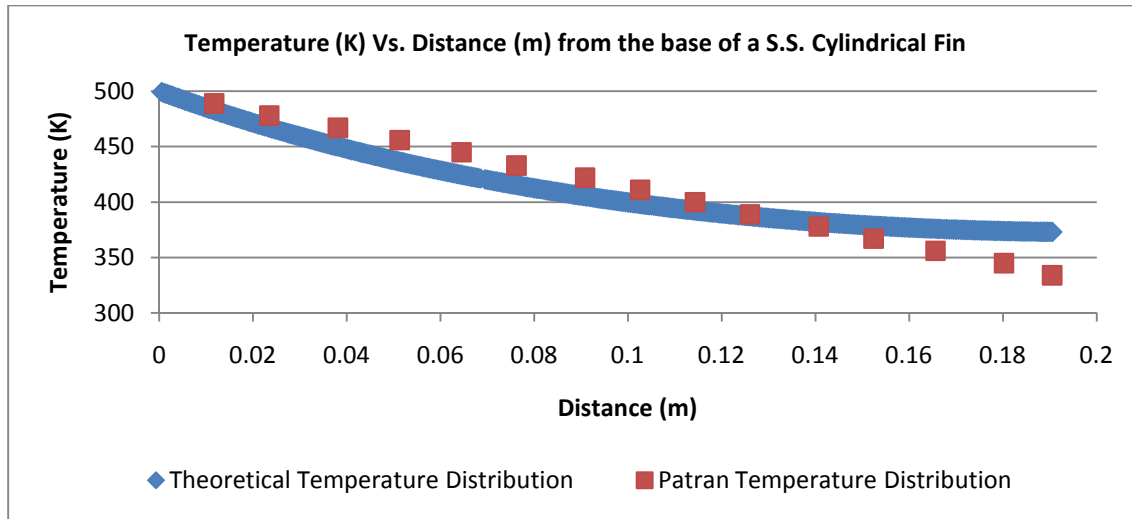


Figure 9. A plot comparing *Patran* results to theoretical for a temperature distribution of a cylindrical fin.

When comparing the results taken from *Patran* to theoretical values, one can see that the software is employed correctly, since the temperature data from an accepted mathematical model matches closely to that of *Patran* results.

8.1. FEA of a Lattice fin

Due to the complex geometry of the Tetralattice™ structure, the part file was cut into a ¼ longitudinally, with interior surfaces insulated so *Patran* could conduct the analysis easier. The overall fin was still 1.5 inches in diameter, and 7.5 inches in length, and having lattice thicknesses of 0.02 inches in diameter, with all the same material properties and boundary conditions applied.

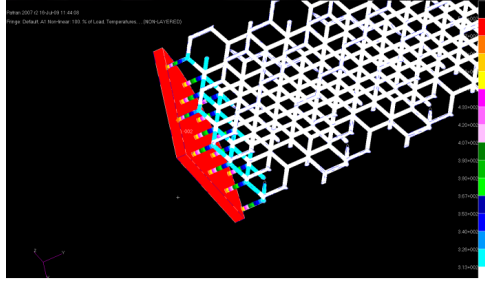


Figure 10. FE Temperature distribution results from Patran for a Uniform Tetralattice™ fin.

When comparing Figure 8 to Figures 10, one can see that *Patran* models the temperature distribution of the simple cylinder as reaching ambient temperature much higher up on the fin as compared to the uniform Tetralattice. However, these results from *Patran* do not depict a true physical representation of the anticipated lattice temperature distribution. A careful examination of the convective boundary condition revealed the reason for this particular temperature distribution. The fin was convecting at the same rate from all surfaces within the interior of the fin to, ambient conditions. This was not an error of the software, but a misunderstanding of the conditions applied to the fin. To explore this error further the author ran simulations of 1 in. tall lattice fins of lattice thicknesses of 0.02 in. and 0.08 in. to see the effect lattices of differing thickness has on heat transfer. The length of 1 in. was to keep the mesh size reasonable for running the simulation and two lattice thicknesses were used to see the effect of lattice thickness on heat transfer.

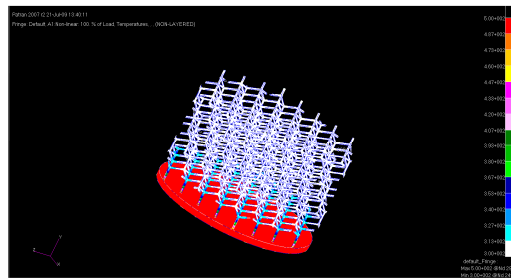


Figure 11. FEA results of lattice fins of 0.02 inch lattice diameters.

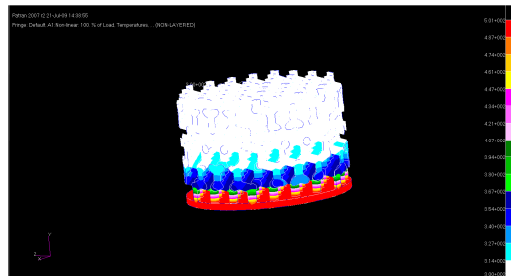


Figure 12. FEA results of lattice fins of 0.08 inch lattice diameters.

When comparing Figure 11 to Figure 12 one can see that the 0.02 thickness fin reaches ambient fin temperature much closer to the base of the fin when compared to the 0.08 thickness fin. With the smaller thickness, there is a greater exposure to the ambient conditions and hence a quicker temperature drop. However these results still are not entirely representative of true conditions. A more accurate model would incorporate the changing thermal properties of air within the fin. A new model was made by filling the negative void within the lattice fin with another solid which will have the thermal properties of air prescribed $k = 0.0263 \left(\frac{W}{m * K} \right)$, so instead of heat flowing directly from the lattice to ambient conditions within the lattice, it is forced to flow throughout the lattice (both metal and air) and then to the outer surface in order to convect to the surroundings from the fin. This would be a more accurate model of the fin, assuming the air within the fin is stagnant. This is a reasonable assumption for natural convection.

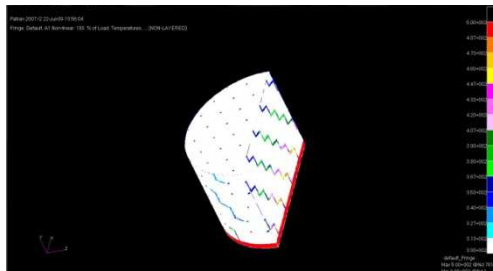


Figure 13. FEA results of the lattice with the stagnant air condition (air in white).

This analysis more accurately models what one would expect the heat transfer of a uniform lattice fin to look like, since heat is drawn further up the lattice, since at the far end of the 1 in. fin the metal only reaches a temperature of 350 K at the center and dropping off slightly as one moves out radially from the center.

9. Conclusion

The refined 1 in. results proved that finite element analysis can be performed on the uniform Tetralattice™ fin. This form of analysis has only been limited by that of hardware used to run *Patran*. The author feels that this analysis performed on the uniform Tetralattice™ fin will prove invaluable in developing an understanding of the thermal properties of the Tetralattice™ structure. In addition this analysis will allow for visualizing and optimizing, the heat transfer properties of Tetralattice™ fins prior to fabrication.

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